OPTIMAL SHOUTING POLICIES OF OPTIONS WITH STRIKE RESET RIGHT

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The reset right embedded in an option contract is defined to be the privilege given to the option holder to reset certain terms in the contract according to specified rules at the moment of shouting, where the time to shout is chosen optimally by the holder. For example, a shout option with strike reset right entitles its holder to choose the time to take ownership of an at-the-money option. This paper develops the theoretical framework of analyzing the optimal shouting policies to be adopted by the holder of an option with reset right on the strike price. It is observed that the optimal shouting policy depends on the time dependent behaviors of the expectation of discounted value of the at-the-money option received upon shouting. During the time period when the theta of the expectation of discounted value of the new option received is positive, it is never optimal for the holder to shout at any level of asset value. At those times when the theta is negative, we show that there exists a threshold value for the asset price above which the holder of a reset put option should shout optimally. For the shout floor options, we obtain an analytic representation of the price function. For the reset put option, we derive the integral representation of the shouting right premium and analyze the asymptotic behaviors of the optimal shouting boundaries at time close to expiry and infinite time from expiry. We also provide numerical results for the option values and shouting boundaries using the binomial scheme and recursive integration method. Accuracy and run time efficiency of these two numerical schemes are compared.

Key Words: reset option, shout floor, optimal shouting policy, numerical scheme

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1. INTRODUCTION

The acute competitions in the markets prompt financial engineers to design option contracts with more exotic features. One such feature is the right given to the holder to reset certain contract terms according to specified rules during the life of the option contract. A simple example is the *reset put option*, where the strike price is reset to be the prevailing asset price at the moment chosen by the holder. The moment to reset is often called the shouting moment. Let X denote the original strike price set at initiation of the option, S_t and S_T denote the asset price at the shouting instant t and maturity date T, respectively. The payoff of the reset put option is given by $\max(X - S_T, 0)$ if no shouting occurs throughout the option's life, and the payoff is modified to $\max(S_t - S_T, 0)$ if shouting occurs at time t before the maturity date T. Upon shouting, the reset put option effectively converts into an at-the-money put option. From the nature of the payoff, it is obvious that the holder would shout only for $S_t > X$ so that an increase in the terminal payoff is resulted after shouting.

Another example is the *shout floor* where the holder can shout at any time t during the life of the contract to install a floor on the return, with the floor value being set at the prevailing asset price S_t at the shouting moment (Cheuk and Vorst, 1997). The terminal payoff of the shout floor is seen to be $\max(S_t - S_T, 0)$ if shouting occurs, but assumes zero value if otherwise. The shout floor can be considered as a special example of a reset put option with the initial strike price set at the zero value.

There exist a wide range of financial instruments with embedded shout features. Gray and Whaley (1999) analyzed the reset feature in the Geared Equity Investment offered by Macquarie Bank. Brenner *et al.* (2000) examined the impact of resetting the terms of previously issued executive stock options on firm performance. Windcliff *et al.* (2001a,b) analyzed the Canadian segregated funds with multiple reset rights on guaranteed level and maturity date. Jaillet *et al.* (2001) studied a special form of shout feature (swing option) that appears in some energy derivative contracts.

In this paper, we develop the linear complimentarity formulation to analyze the optimal shouting policies for options with single shouting right to reset the strike price. Similar to American options with the early exercise right, the pricing of options with the shouting right leads to free boundary value problems. The reward function upon shouting in a reset put option is different from its terminal payoff, a distinctive feature that distinguishes a reset put option from an American option. The optimal shouting policy of a reset put option or a shout floor depends on whether the riskless interest rate r is greater than, equal to or less than the dividend yield q. When $r > q$, it is never optimal for the holder to shout at any asset value at times before some critical time.

This distinctive property of optimal shouting is closely related to the behavior of the theta of the value of an at-the-money put option. Within the time period when the theta of the expectation of the discounted value of the at-the-money put option received upon shouting is positive, it is never optimal for the reset put option holder to shout at any level of asset value. This agrees well with the financial intuition that when the theta assumes positive value, shouting would result in double losses – loss in the value associated with the right to reset and loss in the temporal growth of the value of the at-the-money put to be received. When $r < q$, the expectation of discounted value of the at-the-money put option always decays in value as time proceeds. We prove rigorously that there always exists a threshold value for the asset price above which the holder of the reset put should shout optimally.

Cheuk and Vorst (1997) formulated the pricing model of the shout floor as an optimal stopping problem and developed the lattice schemes for the numerical valuation of single-shout and multi-shout shout floors. No closed form price formulas have been derived in their paper. Windcliff *et al.* (2002a) examined the reset features in Canadian segregated funds through refined numerical procedures. These authors have not performed the theoretical analysis of the characterization of the shouting boundary. In this paper, we provide rigorous analysis of the optimal shouting policies of the shout floors and reset put options. We develop the analytic price formulas and analyze the optimal shouting policies for the shout floors. For the reset put options, we derive the integral representation of the shouting premium and examine the asymptotic behaviors of the optimal shouting boundaries at time close to expiry and infinite time from expiry. We obtain explicitly the limiting critical asset value at infinite time to expiry (for $r < q$) and the threshold time before which it is never optimal to shout (for $r > q$). For the determination of the optimal shouting boundaries, we derive the integral equation for the critical asset value and solve the equation using the recursive integration method. The accuracy and efficiency of the recursive integration method are compared with those of the binomial method.

The paper is organized as follows. In Section 2, we formulate the pricing models for options with the reset feature and examine the time dependent behaviors of the expectation of discounted value of at-the-money puts. Section 3 presents the analytic derivation of the price function and the optimal shouting policies of the shout floor. In Section 4, we explore the characterization of the optimal shouting boundary $S^*(\tau)$ of the reset put option under different conditions on the relative values of the riskless interest rate and the dividend yield. We examine the impact on the shouting policy due to the time dependent behaviors of the expectation of discounted value of at-the-money put option received upon shouting. In particular, the asymptotic behaviors of $S^*(\tau)$ at time close to maturity and infinite time to expiry are examined. The integral representation of the shouting premium of the reset put option is derived. In Section 5, we give the details on the numerical procedure to compute $S^*(\tau)$. The integral equation for the determination of the shouting boundary is developed, which is then solved by the recursive integration method. The accuracy and run time efficiency of the recursive integration method are compared with those of the binomial method. Numerical solutions to the shout floor and reset put options are obtained, which are used to compare with the theoretical results deduced in earlier sections. The paper ends with conclusive remarks in the last section.

2. FORMULATION OF THE PRICING MODELS

We follow the usual Black-Scholes assumptions in the pricing framework for options with the shout feature. In the risk neutral world, the stochastic process for the asset price S is assumed to follow the lognormal diffusion process

$$\frac{dS}{S} = (r-q)dt + \sigma dZ, \tag{2.1}$$

where r and q are the constant riskless interest rate and dividend yield, respectively, σ is the constant volatility and dZ is the standard Wiener process. We let τ denote the time to expiry, where $\tau = T - t$. Here, T is the option expiration date and t is the current time.

2.1 Formulation as free boundary value problems

For either the reset put option or the shout floor, the option becomes an at-the-money put option upon shouting. The price function of this at-the-money put option is seen to be linearly homogeneous in S and takes the form $SP(\tau)$. By setting the strike price to be the current asset price in the Black-Scholes vanilla put option price formula, we obtain

$$P(\tau) = e^{-r\tau}N(-d_2) - e^{-q\tau}N(-d_1), \tag{2.2}$$

where

$$N(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\xi^2/2}d\xi, \quad d_1 = \frac{r-q+\frac{\sigma^2}{2}}{\sigma}\sqrt{\tau} \quad \text{and} \quad d_2 = d_1 - \sigma\sqrt{\tau}. \tag{2.3}$$

The pricing model of the reset put option or the shout floor leads to a free boundary value problem. The linear complementarity formulation of the pricing function $V(S,\tau)$ is given by

$$\begin{aligned} &\frac{\partial V}{\partial \tau} - \frac{\sigma^2}{2}S^2\frac{\partial^2 V}{\partial S^2} - (r-q)S\frac{\partial V}{\partial S} + rV \geq 0, \quad V(S,\tau) \geq SP(\tau), \\ &\left[\frac{\partial V}{\partial \tau} - \frac{\sigma^2}{2}S^2\frac{\partial^2 V}{\partial S^2} - (r-q)S\frac{\partial V}{\partial S} + rV\right][V - SP(\tau)] = 0, \\ &V(S,0) = \begin{cases} \max(X-S,0), & \text{reset put} \\ 0, & \text{shout floor} \end{cases}. \end{aligned} \tag{2.4}$$

The option value becomes the reward function $SP(\tau)$ upon shouting; otherwise, it always stays above $SP(\tau)$. Note that the shout floor corresponds to the reset put option with zero initial strike price. The critical shouting boundary, denoted by $S^*(\tau)$, separates the domain of the problem into the continuation region and stopping region. The shouting boundary is not known *a priori* but has to be solved in the solution procedure of the free boundary value problem. Since the holder shouts only when the asset value reaches sufficiently high level, the continuation region and stopping region are on the left and right hand side of the shouting boundary, respectively. The option price function $V(S,\tau)$ observes the smooth pasting (or "high contact") conditions, that is, continuity of the option value and delta across the optimal shouting boundary $S^*(\tau)$.

In the continuation region, the price function $V(S,\tau)$ satisfies the Black-Scholes equation and $V(S,\tau)$ is above $SP(\tau)$. In the stopping region, the option value becomes the reward function $SP(\tau)$. Substituting into the Black-Scholes equation, we obtain

$$\begin{aligned} &\frac{\partial V}{\partial \tau} - \frac{\sigma^2}{2} S^2 \frac{\partial^2 V}{\partial S^2} - (r-q) S \frac{\partial V}{\partial S} + rV \\ &= \begin{cases} 0 & \text{if } (S,\tau) \text{ lies in the continuation region} \\ Se^{-q\tau} \dfrac{d}{d\tau}[e^{q\tau} P(\tau)] & \text{if } (S,\tau) \text{ lies in the stopping region} \end{cases} . \end{aligned} \tag{2.5}$$

Note that either the continuation region or the stopping region (but not both) can be an empty set.

The tractability of the pricing model for the reset put option and the shout floor stems from the linear homogeneity in S of the reward function. Later, we show that the existence of a non-empty continuation region and/or stopping region depends on the terminal payoff and the time dependent property of the reward function.

2.2 Behaviors of expectation of discounted value of at-the-money puts

Similar to the concept of delayed compensation premium in American option model, the term $Se^{-q\tau} \frac{d}{d\tau}[e^{q\tau} P(\tau)]$ gives the rate of cashflow required to compensate the shout option holder if he does not shout in the stopping region. This term is related to the expectation of the discounted value of the at-the-money put received upon shouting. The sign behaviors of $\frac{d}{d\tau}[e^{q\tau} P(\tau)]$ are summarized in Lemma 2.1.

Lemma 2.1 The function $e^{q\tau} P(\tau)$ observes the following properties.

(i) If $r \leq q$, then it is strictly increasing for $\tau \in (0,\infty)$.

(ii) If $r > q$, there exists a unique critical value $\tau^* \in (0,\infty)$ such that it is strictly increasing for $\tau \in (0,\tau^*)$ and strictly decreasing for $\tau \in (\tau^*,\infty)$.

The proof of Lemma 2.1 is given in Appendix A.

When $\frac{d}{d\tau}[e^{q\tau}P(\tau)] > 0$, the theta (derivative with respect to t) of the expectation of discounted value of the at-the-money put becomes negative. From financial intuition, it is expected that negative theta value is a necessary condition for the commencement of optimal shouting. In other words, the holder should never shout when the theta is positive. The necessary and sufficient conditions for the optimal shouting policies of shout floors and reset put options are explored in the coming sections.

3. ANALYTIC PRICE FORMULA OF THE SHOUT FLOOR

Let $R(S,\tau)$ be the price function of the shout floor. Since there is no strike price X appearing in the terminal payoff function and the obstacle function $SP(\tau)$ observes linear homogeneity in S, one would expect that $R(S,\tau)$ is linearly homogeneous in S. Suppose we write $R(S,\tau) = Sg(\tau)$, and substitute the assumed form of $R(S,\tau)$ into the linear complimentarity formulation (2.4), we obtain the following formulation for $g(\tau)$.

$$
\begin{aligned}
&\frac{d}{d\tau}[e^{q\tau}g(\tau)] \geq 0, \quad g(\tau) \geq P(\tau), \\
&\left\{\frac{d}{d\tau}[e^{q\tau}g(\tau)]\right\}[g(\tau) - P(\tau)] = 0, \\
&g(0) = 0.
\end{aligned}
\tag{3.1}
$$

The well-posedness of the above formulation for $g(\tau)$ justifies the assumption of linear homogeneity of $R(S,\tau)$. We solve for $g(\tau)$ under the following two separate cases:

(i) When $r \leq q$, $\frac{d}{d\tau}[e^{q\tau}P(\tau)]$ is strictly positive for all $\tau > 0$ and $P(0) = 0$; therefore, we can deduce that $g(\tau) = P(\tau), \quad \tau \in (0,\infty)$.

(ii) When $r > q$, we deduce similarly that $g(\tau) = P(\tau)$ for $\tau \in (0,\tau^*]$. When $\tau > \tau^*$, we cannot have $g(\tau) = P(\tau)$. If otherwise, this would lead to $\frac{d}{d\tau}[e^{q\tau}g(\tau)] = \frac{d}{d\tau}[e^{q\tau}P(\tau)] < 0$, a contradiction. Hence, we must have $\frac{d}{d\tau}[e^{q\tau}g(\tau)] = 0$ for $\tau \in (\tau^*,\infty)$. Solving this differential equation and applying the auxiliary condition: $g(\tau^*) = P(\tau^*)$, we obtain $g(\tau) = e^{-q(\tau-\tau^*)}P(\tau^*)$ for $\tau \in (\tau^*,\infty)$. The above results are summarized in Theorem 3.1.

Theorem 3.1 The price function of the shout floor $R(S,\tau)$ has the following analytic representation.

(i) If $r \leq q \qquad R(S,\tau) = SP(\tau), \qquad \tau \in (0,\infty)$ and $S > 0$.

(ii) If $r > q$,

$$R(S,\tau) = \begin{cases} SP(\tau), & \tau \in (0, \tau^*] \text{ and } S > 0 \\ e^{-q(\tau-\tau^*)}SP(\tau^*), & \tau \in (\tau^*, \infty) \text{ and } S > 0, \end{cases}$$

where τ^* is the unique positive root of $\frac{d}{d\tau}[e^{q\tau}P(\tau)]$.

Next, we illustrate that the optimal shouting policy of the shout floor also depends on the sign of $\frac{d}{d\tau}[e^{q\tau}P(\tau)]$. When the sign is non-negative, we have $R(S,\tau) = SP(\tau)$, inferring that the holder should shout at once. This occurs either when (i) $r \leq q$, $\tau \in (0,\infty)$, or (ii) $r > q$, $\tau \leq \tau^*$. Conversely, when $r > q$ and $\tau > \tau^*$, Theorem 3.1 indicates that $R(S,\tau) > SP(\tau)$, so the holder should not shout under such scenario. In summary:

(i) When $r \leq q, S^*(\tau) = 0$ and the continuation region is an empty set for all $\tau \in (0,\infty)$.

(ii) When $r > q$, (a) $S^*(\tau) = 0$ and the continuation region is empty when $\tau \leq \tau^*$, (b) $S^*(\tau) = \infty$ and the stopping region is empty when $\tau > \tau^*$, where τ^* is the root of $\frac{d}{d\tau}[e^{q\tau}P(\tau)]$.

4. OPTIMAL SHOUTING BOUNDARY AND SHOUTING PREMIUM FOR RESET PUT OPTION

Unlike the shout floor, the analytic price formula for the reset put option cannot be obtained. First, we examine the characterization of the optimal shouting boundary $S^*(\tau)$ of the reset put option, in particular, the asymptotic behaviors at $\tau \to 0^+$ and $\tau \to \infty$. Since the new strike upon reset should not be lower than the original strike, we must have $S^*(\tau) \geq X$. We show how the behaviors of $S^*(\tau)$ depend on the relative magnitudes of r and q. We also obtain the integral representation of the shouting premium.

4.1 Asymptotic behaviors of $S^*(\tau)$

For American options, it is well known that the critical asset price at $\tau \to 0^+$ depends on the ratio of r and q. However, this is not so for the reset put option.

Theorem 4.1 The optimal shouting boundary $S^*(\tau)$ for the reset put option starts from X, namely, $S^*(0^+) = X$.

The proof of Theorem 4.1 is presented in Appendix B. Since $Se^{-q\tau}\frac{d}{d\tau}[e^{q\tau}P(\tau)] > 0$ for all S as $\tau \to 0^+$, shouting at any S when $\tau \to 0^+$ would lead to positive gain to the holder. On the other hand, $S^*(0^+)$ must not be less than X. The combination of the two conditions gives $S^*(0^+) = X$.

Next, we examine the asymptotic behaviors of the shouting boundary of the reset put option $S^*(\tau)$ at infinite time to expiry. Let $S^*_{1,\infty}$ denote the limit of $S^*(\tau)$ as $\tau \to \infty$. We would like to show that $S^*_{1,\infty}$ exists when $r < q$, and subsequently determine its corresponding value. This is linked with the existence of the following limit

$$\lim_{\tau\to\infty} e^{r\tau}P(\tau) = \lim_{\tau\to\infty}[N(-d_2) - e^{(r-q)\tau}N(-d_1)] = 1 \quad \text{for} \quad r \le q. \tag{4.1}$$

Let $V(S,\tau)$ be the price function of the reset put option and let $W^\infty(S)$ denote the limit of $e^{r\tau}V(S,\tau)$ as $\tau \to \infty$. The corresponding set of governing equations for $W^\infty(S)$ are given by

$$\frac{\sigma^2}{2}S^2\frac{d^2W^\infty}{dS^2} + (r-q)S\frac{dW^\infty}{dS} = 0, \quad 0 < S < S^*_\infty,$$

$$W^\infty(0) = X, \quad W^\infty(S^*_\infty) = S^*_\infty, \quad \frac{dW^\infty}{dS}(S^*_\infty) = 1. \tag{4.2}$$

This formulation for $W^\infty(S)$ implicitly requires the existence of $\lim_{\tau\to\infty} e^{r\tau}P(\tau)$, so it is applicable only for $r \le q$ [see Dewynne *et al.* (1989)]. The solution to $W^\infty(S)$ is found to be

$$W^\infty(S) = X + \frac{\beta^\beta}{(1+\beta)^{1+\beta}}X^{-\beta}S^{1+\beta}, \quad 0 < S < S^*_\infty, \tag{4.3}$$

where $S^*_\infty = \left(1 + \dfrac{1}{\beta}\right)X$ and $\beta = 2(q-r)/\sigma^2$.

Hence, when $r < q$, $S^*(\tau)$ is defined for $\tau \in (0,\infty)$ with the asymptotic limit $S^*_\infty = \left(1 + \dfrac{1}{\beta}\right)X$. Note that when $r = q$, S^*_∞ becomes infinite.

When $r > q$, we recall that it is never optimal to shout the shout floor at $\tau > \tau^*$; that is, $R(S,\tau) > SP(\tau)$ at $\tau > \tau^*$ when $r > q$. Since $V(S,\tau) \ge R(S,\tau)$ for all S and τ, so when $r > q$, it is never optimal to shout at $\tau > \tau^*$ by virtue of the property: $V(S,\tau) > SP(\tau)$ at $\tau > \tau^*$. We write the critical asset value as $S^*(\tau;X)$ to show its dependence on the strike price X. When $r > q$ and $\tau < \tau^*$, we have shown in Sec. 3.2 that $S^*(\tau;0) = 0$. On the other hand, $S^*(\tau;\infty) = \infty$ since it is never optimal to shout at any asset value when the strike price is infinite. One may expect that $S^*(\tau;X)$ assumes finite value when X is finite, when $r > q$ and $\tau < \tau^*$. The precise statement of the result is stated in Lemma 4.2, and the rigorous proof of which is given in Appendix C.

Lemma 4.2 For $r > q$ and $\tau < \tau^*$, there exists a critical asset price $S^*(\tau)$ such that $V(S,\tau) = SP(\tau)$ for $S \ge S^*(\tau)$.

Note that the compact support property of the terminal payoff of the put and the increasing property of $e^{q\tau}P(\tau)$ for $\tau < \tau^*$ give the sufficiency for the existence of $S^*(\tau)$. With the finiteness property of $S^*(\tau;X)$ for $X \ge 0$, we then have the continuous dependence property of $S^*(\tau;X)$ on X for $X \in [0,\infty)$. Further, we observe that $S^*(\tau;kX) = kS^*(\tau;X)$, where k is a positive constant, thus we can deduce that $S^*(\tau;X)$ is linear homogeneous in X. We now summarize all of the above results as follows:

Theorem 4.3 The behavior of the optimal shouting boundary $S^*(\tau)$ of the reset put option depends on the relative values of r and q.

(i) If $r \leq q$, then $S^*(\tau)$ is finite for $\tau \in (0, \infty)$ and $S^*_\infty = \left(1 + \frac{1}{\beta}\right) X$. In particular, when $r = q$, S^*_∞ becomes infinite value.

(ii) If $r > q$, then $S^*(\tau)$ is finite for $\tau \in (0, \tau^*)$.

4.2 Integral representation of the shouting premium

Let $e(S, \tau)$ denote the shouting premium of the reset put option, and let $\psi(S_\xi; S)$ denote the transition density function for the future asset value S_ξ at ξ periods from now, given the current asset value S. Over the time period $[\xi, \xi + d\xi]$, the present value of the amount of compensation paid to the holder for delayed shouting is given by

$$
\begin{aligned}
&e^{-r\xi} E[S_\xi e^{-q(\tau-\xi)} \frac{d}{du}[e^{qu} P(u)]\Big|_{u=\tau-\xi} 1_{\{S_\xi \geq S^*(\tau-\xi)\}}] \\
&= e^{-r\xi} \int_{S^*(\tau-\xi)}^{\infty} S_\xi e^{-q(\tau-\xi)} \frac{d}{du}[e^{qu} P(u)]\Big|_{u=\tau-\xi} \psi(S_\xi; S)\, dS_\xi
\end{aligned}
\tag{4.4a}
$$

where E is the expectation under the risk neutral measure and $1_{\{\cdot\}}$ is the indicator function, and

$$
\psi(S_\xi; S) = \frac{1}{S_\xi \sigma \sqrt{2\pi\xi}} \exp\left(-\frac{\left[\ln \frac{S_\xi}{S} - \left(r - q - \frac{\sigma^2}{2}\right)\xi\right]^2}{2\sigma^2 \xi}\right). \tag{4.4b}
$$

The shouting premium $e(S, \tau)$ is obtained by summing all these compensations over the whole time interval $[0, \tau]$, and it can be expressed as

$$
\begin{aligned}
e(S, \tau) &= \int_0^\tau e^{-r\xi} \int_{S^*(\tau-\xi)}^{\infty} S_\xi e^{-q(\tau-\xi)} \frac{d}{du}[e^{qu} P(u)]\Big|_{u=\tau-\xi} \psi(S_\xi; S)\, dS_\xi d\xi \\
&= S e^{-q\tau} \int_0^\tau N(d_{1,\tau-u}) \frac{d}{du}[e^{qu} P(u)]\, du,
\end{aligned}
\tag{4.5a}
$$

where

$$
d_{1,\tau-u} = \frac{\ln \frac{S}{S^*(u)} + \left(r - q + \frac{\sigma^2}{2}\right)(\tau - u)}{\sigma\sqrt{\tau - u}}. \tag{4.5b}
$$

Since $S^*(\tau)$ becomes infinite when $\tau > \tau^*$ for $r > q$, one should change the upper integration limit in Eq. (4.5a) from τ to τ^* when $r > q$.

5. NUMERICAL SCHEMES: RECURSIVE INTEGRATION METHOD AND BINOMIAL METHOD

In this section, we derive the integral equation for the determination of the critical asset value $S^*(\tau)$ for the reset put option and illustrate how to use the recursive integration method to solve for the shouting boundary. We then compute the reset put option value by evaluating directly the shouting premium integral. We also evaluate the option value using the binomial method by incorporating the usual dynamic programming procedure of taking the maximum among the continuation value and the reward value upon shouting. The accuracy and run time efficiency of these two methods are illustrated. We also plot the reset put option value and shouting boundaries under different cases of relative magnitudes of r and q.

5.1 Integral equation for $S^*(\tau)$ and recursive integration method

At the critical asset value $S = S^*(\tau), V(S,\tau) = SP(\tau)$. Substituting this relation into Eq. (4.5a), we obtain the following integral equation for $S^*(\tau)$:

$$(5.1a)\quad S^*(\tau)P(\tau) = p_E(S^*(\tau),\tau) + S^*(\tau)e^{-q\tau}\int_0^\tau N(d^*_{1,\tau-u})\frac{d}{du}[e^{qu}P(u)]\ du, \quad \text{for} \quad \tau < \tau^*,$$

where τ^* is taken to be infinite value for $r \le q$. Here, $p_E(S,\tau)$ is the value of the corresponding vanilla put option and

$$(5.1b)\qquad d^*_{1,\xi} = \frac{\ln\frac{S^*(\tau)}{S^*(\tau-\xi)} + \left(r - q + \frac{\sigma^2}{2}\right)\xi}{\sigma\sqrt{\xi}}.$$

We apply the recursive integration method (Huang *et al.*, 1996) to solve for $S^*(\tau)$ from the above integral equation. This is done by integrating the integral premium term using numerical quadrature and determining the optimal shouting boundary $S^*(\tau)$ at discrete instants recursively. Since the integrand function inside the integral term has an integrable square root singularity at $u = 0$, it is necessary to transform the integral into the following form:

$$\int_0^\tau N(d^*_{1,\tau-u})\frac{d}{du}[e^{qu}P(u)]\ du$$

$$(5.2a)\qquad = \int_0^{\sqrt{\tau}} e^{-(r-q)u^2}N(d^*_{1,\tau-u^2})[\sigma n(d_{2,u^2}) - 2u(r-q)N(-d_{2,u^2})]\ du,$$

where

$$(5.2b)\qquad d_{2,\xi} = \frac{\ln\frac{S_\xi}{X} + \left(r - q - \frac{\sigma^2}{2}\right)\xi}{\sigma\sqrt{\xi}} \quad \text{and} \quad d_{1,\xi} = d_{2,\xi} + \sigma\sqrt{\xi}.$$

First, the interval $[0,\sqrt{\tau}]$ is divided into N equally spaced subintervals, with end points $u_i = i\Delta u, i = 0,1,\cdots,N$, where $\Delta u = \sqrt{\tau}/N$. We define the function

$$(5.3)\qquad f(S^*(\tau),S^*(\eta);\tau,\eta) = e^{-(r-q)\eta}N(d^*_{1,\tau-\eta})[\sigma n(d_{2,\eta}) - 2(r-q)\sqrt{\eta}N(-d_{2,\eta})],$$

and write $\eta_i = u_i^2$ and $S_i^* = S^*(\xi_i)$, then the recursive scheme for the determination of S_i^* is given by

$$S_i^* P(S_i^*) = p_E(S_i^*, \eta_i) + \frac{\Delta u}{2} S^*(\eta_i) e^{-q\eta_i}$$

$$\left[f(S_i^*, S_0^*; \eta_i, \eta_0) + f(S_i^*, S_i^*; \eta_i, \eta_i) + 2\sum_{k=1}^{i-1} f(S_i^*, S_k^*; \eta_i, \eta_k) \right], \quad i = 1, 2, \cdots, N. \tag{5.4}$$

5.2 Numerical calculations and comparison of performance of numerical schemes

First, we would like to use the binomial method to price the shout floor and compare the numerical results with those obtained from the analytic price formula given in Theorem 3.1. Let Δt be the time step, u and d be the upward and downward jump ratios, respectively, in the binomial tree, where $u = 1/d = e^{\sigma\sqrt{\Delta t}}$. Let p be the probability of the upward jump, where $p = \dfrac{e^{(r-q)\Delta t} - d}{u - d}$. Let V_j^n denote the numerical approximation to the shout floor value $R(Su^j, T - n\Delta t)$. The binomial scheme for pricing the shout floor is given by the following dynamic programming procedure:

$$V_j^n = \max(Su^j P(T - n\Delta t), e^{-r\Delta t}[pV_{j+1}^{n+1} + (1-p)V_{j-1}^{n+1}]$$

$$j = -n, -n+2, \cdots, n, \text{ and } n = 0, \cdots, N-1, \tag{5.5}$$

where N is the total number of time steps. For the shout floor, the terminal payoff is given by $V_j^N = 0, j = -N, -N+2, \cdots, N$. The reset put can be priced in exactly the same manner, except that the terminal payoff is modified to $V_j^N = \max(X - Su^j, 0), j = -N, N+2, \cdots, N$.

In Table 1, we list the values of shout floors with varying values of r and q obtained from the binomial method and analytic formulas. The other parameter values used in the calculations are: $S = X = 100, \sigma = 0.2$ and $\tau = 5$. We observe that the binomial calculations give highly accurate results even with small number of time steps, N. The various values of the critical time τ^* are also listed in the table. When $r \leq q, \tau^*$ does not exist; and for convenience, we take τ^* to assume infinite value. At those times where $\tau < \tau^*$, according to the results obtained in Theorem 3.1, the holder should shout the shout floor at once at any asset value. Hence, the shout floor value is equal to the value of the at-the-money put. On the other hand, when $\tau > \tau^*$, the holder waits until τ falls to τ^* in order that it is optimal to shout. In this case, the shout floor value is higher than the value of the at-the-money put. The numerical results reported in Table 1 verify all of the above theoretical predictions.

In Table 2, we demonstrate the comparison of the numerical accuracy and run time efficiency of the binomial method and the recursive integration method for pricing the reset put option. The values of the reset put options are obtained with varying values of r, q, σ and X. Since there is no closed form analytic formula available for the reset put option, we ran the binomial calculations with 50000 time steps and considered the numerical results obtained as the exact solution of the option value. For the binomial calculations, we accelerated the rate of convergence by using extrapolation techniques. Assuming linear rate of the convergence, we obtained the extrapolated value with $N = 1000$ by adding to the numerical solution the difference of the computed values obtained with $N = 1000$ and $N = 500$. Both the binomial method and the recursive integration method give highly accurate results. The Root Mean Squared Errors (RMSE) shown in Table 2 are obtained by taking the square root of the average of the squares of errors of option values computed at varying asset values and values of time to expiry. The extrapolated binomial method requires 1000 time steps in order to achieve the same level of numerical accuracy as that acheived by the recursive integration method with 64 time steps. Though the algorithmic design of the recursive integration method is more elaborate, the CPU time required for the recursive integration method is only about 7% that of the binomial calculations for comparable level of numerical accuracy.

In Table 3, we show more detailed comparison of accuracy and run time efficiency of the binomial method and the recursive integration method with varying number of time steps. When the number of time steps in the binomial scheme is doubled, the RMSE is roughly halved, indicating an approximate linear rate of convergence of the binomial scheme. The results shown in Table 3 illustrate that the application of the extrapolation procedure to the binomial scheme does improve the level of accuracy of the binomial scheme. The CPU time required for both numerical methods increases roughly four-fold when the number of time steps is doubled. For a given level of numerical accuracy, the run time efficiency of the recursive integration method always wins over that of the binomial method (even with extrapolation).

5.3 Pricing behaviors and optimal shouting boundaries

We applied the recursive integration method to determine the option value $V(S, \tau)$ and the optimal shouting boundary $S^*(\tau)$ of the reset put option. In all calculations, we take the strike price $X = 1.0$ and volatility $\sigma = 20\%$.

FIGURES 1a and 1b show the plots of $V(S, \tau)$ against S at different values of τ, corresponding to $r < q$ and $r > q$, respectively. The price functions $V(S, \tau)$ show no monotonic property in τ. This behavior is in contrast to the American options where American option price functions are always monotonically increasing in τ. The lack of monotonicity in τ may be attributed to the fact that the derivative received upon shouting is an at-the-money European put option, and the price function of a European put option does not exhibit monotonicity in τ. For $r < q$, each price curve touches tangentially the line representing the value of the corresponding at-the-money put option (see FIGURE 1a). When $r > q$, there exists a critical value of τ above which it is never optimal to shout. When the following set of parameter values are used in the option model: $r = 0.06, q = 0.02, \sigma = 0.2$ and $X = 1$, this critical value of τ is found to be 5.7121. In FIGURE 1b, we observe that when $\tau < 5.7121$ (say, $\tau = 0.5$ or $\tau = 1.5$), the price curve touches the line representing the value of the at-the-money put. However, when $\tau > 5.7121$ (say, $\tau = 6.0$), the price curve always stays above the at-the-money put value line.

In FIGURES 2a, 2b and 2c, we plot the critical asset price $S^*(\tau)$ as a function of τ corresponding to $r < q, r = q$ and $r > q$, respectively. Firstly, when $r < q$ (see FIGURE 2a), $S^*(\tau)$ is defined for $\tau \in (0, \infty)$ and $\lim_{\tau \to \infty} S^*(\tau) = 1.5$. This asymptotic value agrees with $S^*(\tau)$ as given in Theorem 4.3. Secondly, when $r = q$ (see FIGURE 2b), $S^*(\tau)$ tends to infinity as τ tends to infinity. Lastly, when $r > q$ (see FIGURE 2c), $S^*(\tau)$ is defined only for $\tau \in (0, \tau^*)$, where τ^* is the root of $e^{q\tau}P(\tau)$. Such behavior of $S^*(\tau)$ indicates that it is never optimal to shout when $\tau > \tau^*$. In all cases, $S^*(\tau)$ is a monotonically increasing function of τ.

6. CONCLUSION

The shout feature embedded in a derivative entitles the holder the right to reset certain terms in the derivative contract. This may be interpreted as the privilege given to the holder to convert the original derivative to a new derivative. Since the critical asset price at which the holder optimally shouts is not known a prior but has to be determined in the solution process, the pricing models are formulated as free boundary value problems.

For both shout floors and reset put options, we show that the optimal shouting policies depend on the time decay behaviors of the expectation of discounted value of the at-the-money option received upon shouting. The behaviors of the optimal shouting boundaries of the reset put options depend crucially on the relative values of the riskless interest rate r and dividend yield q. When $r \leq q$, the shouting boundary of the reset put option is defined at all times. This implies that at any time during the life of the option, the holder should choose to shout optimally when the asset value rises to some threshold value. On the other hand, when $r > q$, there exist a critical time before which it is never optimal for the holder to shout the reset put option at any asset value level. When $r \leq q$, the shout floor should be shouted at once at any time and at any asset price level. When $r > q$, there exists a critical time before which it is never optimal for the holder to shout the shout floor. Upon reaching the critical time, the shout floor should be shouted at once.

Several analytic formulas have been derived in the paper. We obtain the closed form price formula of the shout floor and the integral representation of the shouting premium of the reset put option. From the integral representation of the shouting premium, we derive the integral equation for the determination of the shouting boundary and compute the value of the reset put option using the recursive integration method. We also compute the values of the shout floors and reset put options using the binomial method. Both the binomial method and the recursive integration method give highly accurate results for the price functions. The algorithmic design of the binomial method is simpler than that of the recursive integration method. However, the recursive integration method requires less CPU time to achieve the same level of accuracy as that obtained from the binomial method.

The main contribution of our paper lies on the theoretical analysis of the characterization of the optimal shouting boundary of the reset put options. Such analyses are made possible, thanks to the linear homogeneity property of the price functions of at-the-money put options. With the initial strike price set at zero in a shout floor, we are able to solve for its price function completely. For future works, we may consider reset put options with multiple shouting rights and rights to reset on both the strike price and maturity date of the option contract.

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APPENDIX A: PROOF OF LEMMA 2.1

Since $e^{q\tau}P(\tau;r,q) = P(\tau;r-q,0)$, it suffices to consider the sign behavior of $\dfrac{d}{d\tau}[P(\tau;r,0)]$, where

$$P(\tau;r,0) = e^{-r\tau}N(-d_2) - N(-d_1), \quad \alpha = \frac{r-\frac{\sigma^2}{2}}{\sigma}, \quad d_2 = \alpha\sqrt{\tau}, \quad d_1 = d_2 + \sigma\sqrt{\tau}.$$

The derivative of $P(\tau;r,0)$ is found to be

$$\frac{d}{d\tau}P(\tau;r,0) = e^{-r\tau}\left[-rN(-d_2) + \frac{\sigma}{2\sqrt{\tau}}n(-d_2)\right].$$

We write $f(\tau) = -rN(-d_2) + \dfrac{\sigma}{2\sqrt{\tau}}n(-d_2)$.

(a) When $r \le 0$, we always have $f(\tau) > 0$ so that $\dfrac{d}{d\tau}P(\tau;r,0) > 0$.

(b) When $r > 0$, we examine the sign behavior of $\dfrac{d}{d\tau}P(\tau;r,0)$ by considering the property of

$$f'(\tau) = \frac{\sigma n(-d_2)}{4\sqrt{\tau}}\left[\alpha(\alpha+\sigma) - \frac{1}{\tau}\right].$$

When $\alpha > 0$, $f'(\tau)$ has a unique root $\widehat{\tau} = \dfrac{1}{\alpha(\sigma+\alpha)}$. The function $f(\tau)$ has its absolute minima at $\tau = \widehat{\tau}$. Together with $f(0^+) \to \infty$ and $f(\infty) < 0$, we conclude that $f(\tau)$ has exactly one root in $(0,\infty)$. When $\alpha \le 0$, $f'(\tau) < 0$ for $\tau \in (0,\infty)$; so $f(\tau)$ also has exactly one root in $(0,\infty)$.

APPENDIX B: PROOF OF THEOREM 4.1

Let $D(S,\tau)$ denote the difference between the values of the reset put option and its corresponding at-the-money put option. Note that $D(S,\tau) \ge 0$ for all S and τ. In the continuation region, $D(S,\tau)$ is governed by

$$\frac{\partial D}{\partial \tau} - \frac{\sigma^2}{2}S^2\frac{\partial^2 D}{\partial S^2} - (r-q)S\frac{\partial D}{\partial S} + rD = -S[P'(\tau) + qP(\tau)], \quad 0 < S < S^*(\tau), \tau > 0.$$

As $\tau \to 0^+$, we observe that $-S[P'(\tau)+qP(\tau)] \to -\infty$. Assume the contrary, suppose $S^*(0^+) > X$, and consider $S \in (X, S^*(0^+))$, we have $D(S,0^+) = 0$ so that

$$\frac{\partial D}{\partial \tau}(S,0^+) = -S[P'(0^+) + qP(0^+)] < 0.$$

This would imply $D(S,0^+) < 0$, a contradiction to $D(S,\tau) \ge 0$ for all τ. Therefore, we must have $S^*(0^+) \le X$. On the other hand, since the reset strike must not be lower than the original strike, we then have $S^*(0^+) \ge X$. Combining the results, we obtain $S^*(0^+) = X$.

APPENDIX C: PROOF OF LEMMA 4.2

The idea of the proof stems from Brezis and Friedman (1976). Let $D(S,\tau) = V(S,\tau) - SP(\tau)$, which is monotonically decreasing with S. Therefore, it suffices to show that for $r > q$ and $\tau < \tau^*$, $D(S,\tau) = 0$ when S is sufficiently large. We apply the transformation $D(S,\tau) = Se^{-q\tau}\widetilde{D}(x,\tau)$ and $S = e^x$ to obtain

$$\frac{\partial \widetilde{D}}{\partial \tau} - \frac{\sigma^2}{2}\frac{\partial^2 \widetilde{D}}{\partial x^2} - (r - q + \frac{\sigma^2}{2})\frac{\partial \widetilde{D}}{\partial x} + (r-q)\widetilde{D} \geq -\frac{d}{d\tau}[e^{q\tau}P(\tau)], \quad \widetilde{D}(x,\tau) \geq 0,$$

$$\left[\frac{\partial \widetilde{D}}{\partial \tau} - \frac{\sigma^2}{2}\frac{\partial^2 \widetilde{D}}{\partial x^2} - (r - q + \frac{\sigma^2}{2})\frac{\partial \widetilde{D}}{\partial x} + (r-q)\widetilde{D} + \frac{d}{d\tau}[e^{q\tau}P(\tau)]\right]\widetilde{D}(x,\tau) = 0,$$

$$\widetilde{D}(x,0) = \max(Xe^{-x} - 1, 0).$$

Note that $\widetilde{D}(x,0)$ has compact support. We now construct an auxiliary function $\omega(x)$ as follows:

$$\omega(x) = \begin{cases} \gamma(R_0 - x)^2, & x \leq R_0 \\ 0, & x > R_0, \end{cases}$$

where the parameters R_0 and γ are to be determined. Note that for $x \leq R_0$

$$\begin{aligned} &\left[\frac{\partial}{\partial \tau} - \frac{\sigma^2}{2}\frac{\partial^2}{\partial x^2} - \left(r - q + \frac{\sigma^2}{2}\right)\frac{\partial}{\partial x} + (r-q)\right](R_0 - x)^2 \\ = &-\sigma^2\gamma + 2(r - q + \frac{\sigma^2}{2})(R_0 - x) + (r-q)(R_0 - x)^2 \geq -\sigma^2\gamma, \end{aligned}$$

so that

$$\left[\frac{\partial}{\partial \tau} - \frac{\sigma^2}{2}\frac{\partial^2}{\partial x^2} - \left(r - q + \frac{\sigma^2}{2}\right)\frac{\partial}{\partial x} + (r-q)\right]\omega(x) = \begin{cases} -\sigma^2\gamma, & x \leq R_0 \\ 0, & x > R_0. \end{cases}$$

By Lemma 2.1 (ii), $\frac{d}{d\tau}[e^{q\tau}P(\tau)]$ is positive over $(0,\tau^*)$. We can always find R_0 sufficiently large and γ sufficiently small such that

$$\omega(\ln X) \geq \widetilde{D}(\ln X, \tau)$$

and

$$-\sigma^2\gamma \geq -\frac{d}{d\tau}[e^{q\tau}P(\tau)] \quad \text{for} \quad x \geq \ln X.$$

We then use the comparsion principle to infer that $\omega(x) \geq \widetilde{D}(x,\tau)$ for $x \geq \ln X$, thus $\widetilde{D}(x,\tau) = 0$ for $x \geq R_0$. This is the desired result.

		value of shout floor				critical time	value of at-the-money put
		binomial method			analytic		
r	q	$N = 25$	$N = 50$	$N = 100$	price	τ^*	
0.06	0	6.0259	6.0263	6.0264	6.0264	2.94	5.6968
	0.03	8.9487	8.9487	8.9487	8.9487	8.91	8.9487
	0.06	13.1078	13.1078	13.1078	13.1078	∞	13.1078
	0.12	23.4209	23.4209	23.4209	23.4209	∞	23.4209
0.1	0	3.7737	3.7737	3.7737	3.7737	1.20	2.2684
	0.03	4.5123	4.5125	4.5128	4.5128	2.26	3.9464
	0.06	6.3537	6.3537	6.3537	6.3537	5.71	6.3537
	0.12	13.3571	13.3571	13.3571	13.3571	∞	13.3571

Table 1 The values of the shout floors are computed using the binomial method with varying number of time steps, N. The parameter values used in the calculations are: $S = X = 100, \sigma = 0.2$ and $\tau = 5$. The accuracy of the binomial calculations are compared with those obtained using the analytic price formulas (see Theorem 3.1). The various values of the critical time τ^* are also listed. The shout floor value equals the value of the at-the-money put when $\tau < \tau^*$.

σ	r	q	X	binomial method			recursive integration N = 64
				N = 50000	N = 1000	extrapolated N = 1000	
0.1	0.06	0.03	95	3.7974	3.7971	3.7972	3.7975
			100	4.5124	4.5118	4.5120	4.5125
			105	5.4995	5.4988	5.4990	5.4996
0.1	0.03	0.06	95	15.3583	15.3570	15.3571	15.3605
			100	17.1770	17.1760	17.1763	17.1764
			105	19.7754	19.7747	19.7749	19.7752
0.2	0.06	0.03	95	12.3779	12.3776	12.3776	12.3782
			100	13.5807	13.5799	13.5803	13.5810
			105	14.9688	14.9684	14.9687	14.9691
0.2	0.03	0.06	95	24.4384	24.4376	24.4377	24.4382
			100	26.4197	26.4187	26.4191	26.4199
			105	28.7275	28.7269	28.7272	28.7278
0.3	0.06	0.03	95	21.8264	21.8261	21.8264	21.8270
			100	23.3167	23.3156	23.3161	23.3173
			105	24.9434	24.9426	24.9426	24.9440
0.3	0.03	0.06	95	34.1756	34.1746	34.1715	34.1762
			100	36.2954	36.2940	36.2946	36.2962
			105	38.6219	38.6207	38.6209	38.6227
RMSE				-	3.7e-3	2.6e-3	2.9e-3
CPU time (sec)				-	1.88	2.35	0.17

Table 2 Comparison of accuracy and run time efficiency of the binomial method (with and without extrapolation) and the recursive integration method for pricing the reset put otpion. Other parameter values used in the calculations are: $S = 100, \tau = 5$. The solution obtained with 50000 time steps is considered to be "exact". The accuracy of the extrapolated binomial method with 1000 time steps is comparable to that of the recursive integration method with 64 time steps. However, the CPU time required for the recursive integration method is only about 7% that of the binomial calculations.

number of time steps (binomial; recursive)	binomial method				recursive integration method	
	without extrapolation		with extrapolation			
	RMSE	CPU (sec)	RMSE	CPU (sec)	RMSE	CPU (sec)
1000; 64	3.7e-3	1.88	2.6e-3	2.35	2.9e-3	0.17
2000; 128	1.7e-3	7.6	1.0e-3	9.3	8.5e-4	0.63
4000; 256	7.4e-4	30	4.5e-4	37	3.7e-4	2.7
8000; 512	3.3e-4	123	1.9e-4	171	1.6e-4	8.7

Table 3 This table illustrates the effect of increasing number of time steps on the accuracy improvement and run time increment of the binomial method (with and without extrapolation) and the recursive integration method for pricing the reset put option.

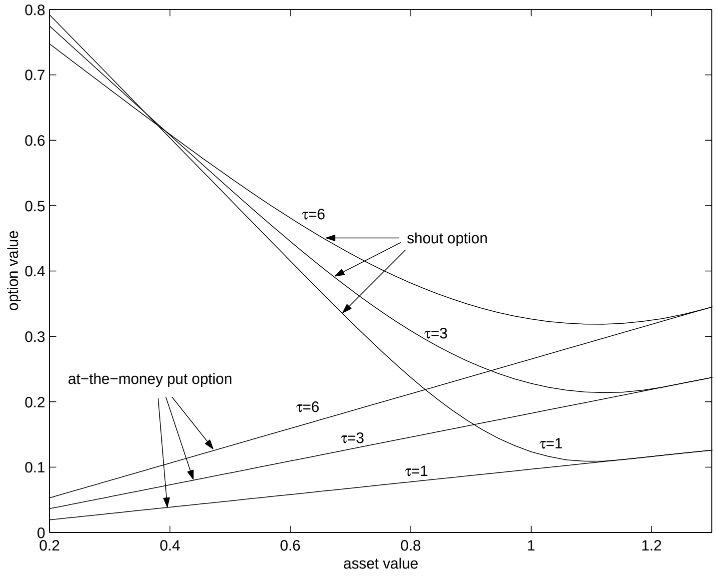

FIGURE 1a. Plot of the value of the reset put option against the asset value for $r < q$ at different values of time to expiry, τ. The parameter values used in the calculations are: $r = 0.02, q = 0.06, \sigma = 0.2$ and $X = 1.0$. Each price curve touches tangentially the line representing the value of the corresponding at-the-money put option.

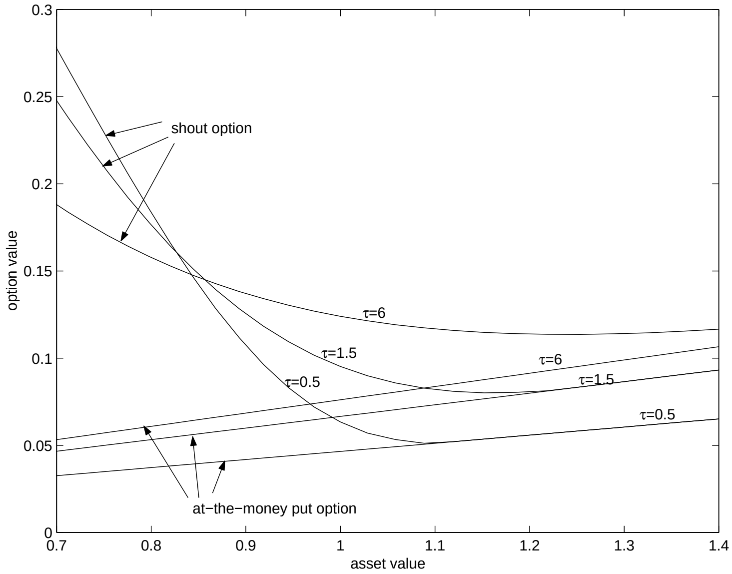

FIGURE 1b. Plot of the value of the reset put option against the asset value for $r > q$ at different values of time to expiry, τ. The parameter values used in the calculations are: $r = 0.06, q = 0.02, \sigma = 0.2$ and $X = 1.0$. The critical value of time to expiry beyond which it is never optimal to shout is found to be 5.7121. The price curve corresponding to $\tau = 6$ (which is greater than 5.7121) never touches the line representing the value of the at-the-money put option.

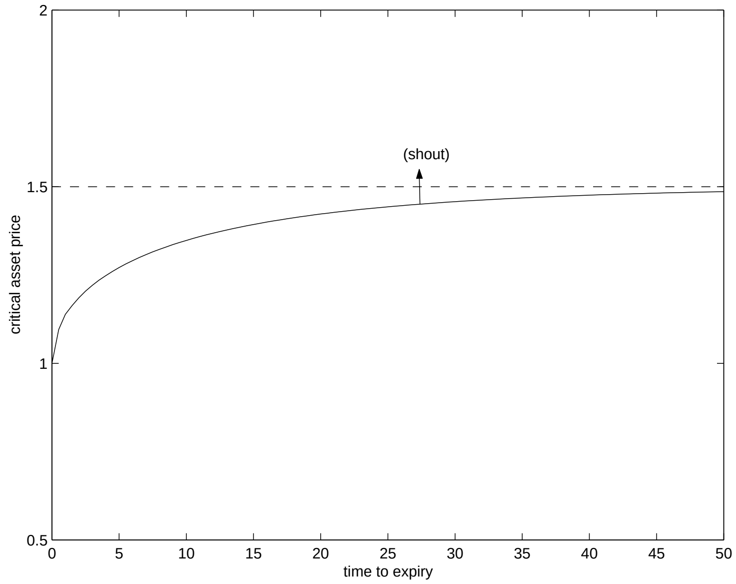

FIGURE 2a. Plot of the shouting boundary of the reset put option as a function of time to expiry for $r < q$. The parameter values used in the calculations are: $r = 0.02, q = 0.06, \sigma = 0.2$ and $X = 1.0$. The asymptotic value of the critical asset price at infinite time to expiry is found to be 1.5.

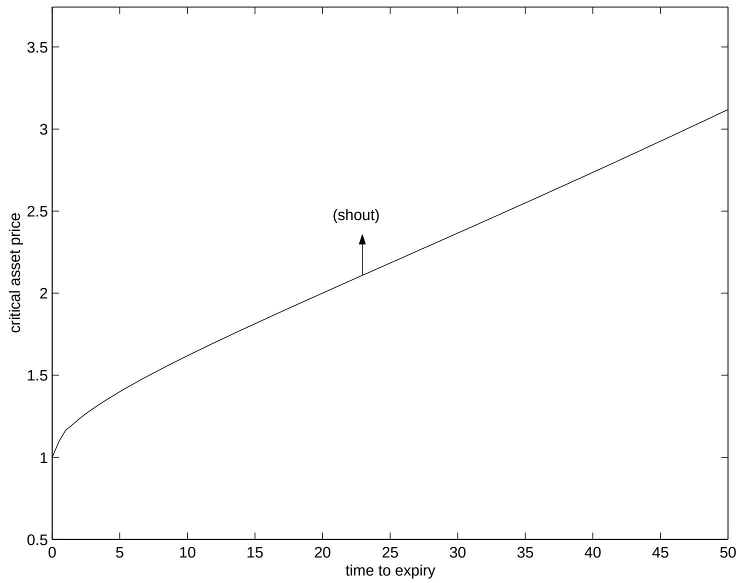

FIGURE 2b. Plot of the shouting boundary of the reset put option as a function of time to expiry for $r = q$. The parameter values used in the calculations are: $r = 0.06, q = 0.06, \sigma = 0.2$ and $X = 1.0$. The critical asset price increases monotonically with increasing time to expiry and tends to infinity at infinite time to expiry.

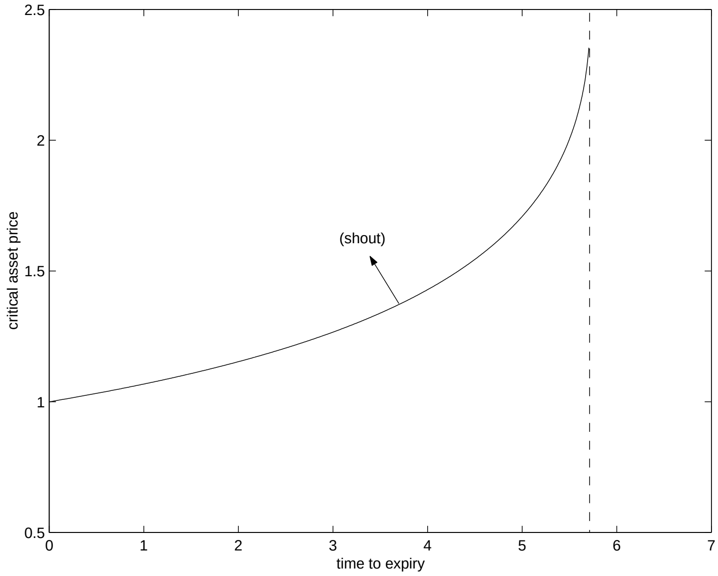

FIGURE 2c. Plot of the shouting boundary of the reset put option as a function of time to expiry for $r > q$. The parameter values used in the calculations are: $r = 0.06, q = 0.02, \sigma = 0.2$ and $X = 1.0$. The critical value of the time to expiry beyond which it is never optimal to shout is found to be 5.7121.